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LETTER TO THE EDITOR

The equation curl $\boldsymbol{B} = k\boldsymbol{B}$ and magnetic fields

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Abstract. In two recent letters Salingaros has put forward physical arguments to claim that solutions of curl $\boldsymbol{B} = k\boldsymbol{B}$ cannot represent real magnetic fields. However, the first part of his assertions is only a restatement of the well known result that this equation cannot be satisfied everywhere over a non-vanishing magnetic field. However, it does not rule out the use of the equation locally over a restricted region of the field. The second part suffers from an incorrect assumption which invalidates his conclusion.

In two recent communications, Salingaros (1986a, b) has claimed that a solution of the equation

$$\text{curl}\ \boldsymbol{B} = k\boldsymbol{B} \qquad k = \text{constant} \neq 0 \tag{1}$$

cannot represent the magnetic field $\boldsymbol{B}$ in a real physical situation. In the first of these letters, he questioned the transformation and symmetry properties of equation (1). In reply, we showed that there were errors in the equations used by Salingaros and pointed out that there was no theoretical constraint which prevented the representation of magnetic fields by solutions of equation (1) locally over a restricted region of a plasma (Maheswaran 1986).

In his second letter, Salingaros (1986b) has used qualitative physical arguments to claim that solutions of equation (1) have no physical relevance. Though not precisely stated, he seems to claim that curl $\boldsymbol{B}$ and, therefore, the current density $\boldsymbol{j}$, cannot be parallel to $\boldsymbol{B}$ *anywhere* in a conductor. This, he has not established. His contention is based on two considerations, which are as follows.

(i) That forces must be present to drive electrical currents and, therefore, $\boldsymbol{j}\wedge\boldsymbol{B}$ cannot vanish. Hence, curl $\boldsymbol{B}$ cannot be parallel to $\boldsymbol{B}$.

(ii) That, when an ambient magnetic field $\boldsymbol{B}_0$ and a parallel current $\boldsymbol{j}$ are present, there will be a generated field $\boldsymbol{B}_j$, which cannot be parallel to $\boldsymbol{j}$. Thus, the resultant magnetic field is not parallel to $\boldsymbol{j}$.

We shall examine each of these assertions. The first assertion that forces must be present in order to drive currents can be taken to be only a restatement of the well known result that a non-vanishing magnetic field cannot satisfy equation (1) everywhere (e.g. Ferraro and Plumpton 1966), i.e. there must be regions where the magnetic force $\boldsymbol{j}\wedge\boldsymbol{B}$ does not vanish. But, this does not preclude the possibility that, in some restricted region within a plasma, the magnetic field $\boldsymbol{B}$ could be parallel to the current density $\boldsymbol{j}$ so that the magnetic force vanishes. This is likely to be in a region where there is no external force strong enough to balance the magnetic force.

In the second assertion, Salingaros claims that curl $\boldsymbol{B}_j$ cannot be parallel to $\boldsymbol{B}_j$, i.e. he has built in the assumption that equation (1) applied to $\boldsymbol{B}_j$ has no solutions. However, we know that this equation has solutions (Ferraro and Plumpton 1966) and, therefore, both his assumption and conclusion are incorrect. Also, Brownstein (1987) has recently pointed out that $\boldsymbol{B}$ need not be locally orthogonal to $\boldsymbol{j} = \text{curl}\ \boldsymbol{B}$, contradicting Salingaros. More importantly, it is the fact that there are solutions with $\boldsymbol{B}$ parallel to $\boldsymbol{j}$ which enables us to construct physical models in which the magnetic force $\boldsymbol{j}_\wedge \boldsymbol{B}$ vanishes locally. Moreover, we believe that splitting up the magnetic field into two parts called the ambient field and the field generated by the current $\boldsymbol{j}$ is misleading and cannot be part of a rigorous proof. In a problem of this nature, one should consider the totality of the electromagnetic field in which the resultant vectors $\boldsymbol{E}$, $\boldsymbol{j}$ and $\boldsymbol{B}$ are related through Maxwell's equations, Ohm's law and appropriate boundary conditions.

Solutions of Maxwell's equations which also satisfy equation (1) are known (e.g. Ferraro and Plumpton 1966). There is no reason why magnetic fields corresponding to some of these solutions may not be realised within a restricted region in a plasma, when the field is appropriately set up in the other regions.

References

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